

JOHANNES KEPLER (December 27, 1571 – November 15, 1630)

by HEINZ KLAUS STRICK, Germany

In 1609, more than 400 years ago, JOHANNES KEPLER's book entitled *Astronomia nova* (New Astronomy) was published in Prague.

It contained two laws on the motion of the planets, which were held in their orbit around the sun by a *magnetic force*. Ten years later he added another law to it.

KEPLER himself never referred to them as *laws*.

- (1) The planets move on elliptical orbits with the sun at one focal point.
- (2) The connecting line between the sun and the planet sweeps out areas of equal size in equal periods of time.
- (3) The squares of the orbital times of the planets are proportional to the third power of the average distance from the sun.



For this deeply religious man, the world was created by God according to a mathematical plan (a similar idea can already be found in PLATO and PYTHAGORAS). KEPLER's research was solely concerned with understanding God's creation, and the *laws* he discovered were, in his opinion, only an expression of world harmony (*Harmonices mundi* - title of the work from 1618):

I feel myself taken in an unspeakable rapture by the divine spectacle of heavenly harmony. For we see here how God, like a human architect, has approached the foundation of the world according to order and rule.

JOHANNES KEPLER came from a simple background. His father, a mercenary, left the family when the child was five years old and his mother ran his grandfather's inn. The sickly boy was allowed to attend a Latin school in Leonberg as well as convent schools in Adelberg and Maulbronn and at the age of 18 he studied Protestant theology at the University of Tübingen.

Mathematics and astronomy were normal components of the study of theology, so it is through his mathematics teacher MICHAEL MAESTLIN that he learnt of the new world view of the Canon NIKOLAUS COPERNICUS and was soon convinced of its correctness.





Because of his views, which were not always conformist, the young theologian had no chance of an academic career at a university dominated by orthodox Lutherans. At the age of 23, for example, he switched to the Protestant Stift School in Graz as a teacher of mathematics and morals. At the same time he prepared astrological forecasts. He became famous when, by accident, he correctly predicted a cold winter and the invasion of Turkish armies in a horoscope.

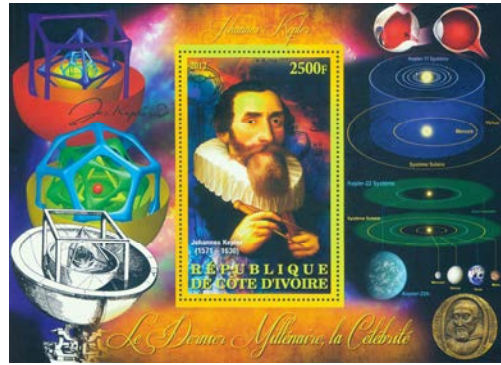
His fame as a successful astrologer contributed to the publication in 1597 of a paper entitled *Mysterium Cosmographicum* (The World Mystery), in which he described the Sun at the centre of a system of six planets. KEPLER was convinced that the sun had six planets (according to the knowledge of the time) and that there were exactly five PLATONIC bodies, which were part of a divine plan. The ratios of the orbital radii determined by COPERNICUS led him to a mathematical model in which six spheres and the five PLATONIC bodies were nested within each other: Saturn's orbit was on a sphere that enclosed a cube (hexahedron), which in turn contained a sphere on which the orbit of Jupiter lay; a tetrahedron fitted into this sphere, on whose internal sphere Mars moved. Then came the dodecahedron, octahedron and icosahedron, each enclosing spheres with the orbits of Earth, Venus and Mercury.



KEPLER explained deviations of the orbits from the circular form (which was still accepted at that time) with a certain thickness of the spherical shells. The work ended with the praise of God, who had granted him the grace of this insight into His creation. He sent his work to GALILEI, among others, with whom he remained in contact in the following years.

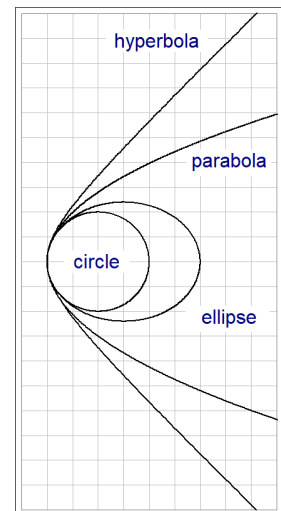
In the same year he married BARBARA MÜHLECK, daughter of a miller and in the 14 happy years of marriage four children were born, of which only two survived. In 1600, as part of a wave of counter-reformation, he was expelled from Catholic Graz, but then found a job as mathematics assistant to the astronomer TYCHO BRAHE in Prague. The two complemented each other happily: BRAHE was a brilliant observer - the theoretician KEPLER a brilliant mathematician.





However, both presented different views of the world: BRAHE also recognised that the world view of PTOLEMY was no longer tenable, but he developed his own model, which stood "between" that of COPERNICUS and that of PTOLEMY: In the centre of the system was the earth and the moon and sun orbited around it. The (other) planets revolved around the sun. A strange rivalry developed between the two: on the one hand BRAHE hoped that his theories could be confirmed by KEPLER's mathematical abilities, but on the other hand he feared that the latter might use his data for his own research and therefore he gave him only a limited insight into his documents.

When BRAHE died under unexplained circumstances, KEPLER became his successor as Imperial Court Mathematician (and Court Astrologer) and was finally able to use all the observational data. In the orbit of Mars he found deviations from a circular orbit of (only) eight minutes of arc, but correctly concluded from this that Mars did not move on a circular orbit but on an elliptical orbit around the Sun (and the other planets did so as well). He further developed the conic section theory of the APOLLONIUS OF PERGA (260 – 190 BC) and was the first to describe conic sections with the aid of focal point equations. He recognised that parabolas could be understood as the boundary cases of ellipses and hyperbolas.



In 1611 KEPLER had to move again for religious reasons. From Linz he mapped Upper Austria, wrote a book on the geometry of optics (*Dioptrice*) and explained how the path of light rays could be traced in optical instruments. He also designed a new telescope in which he used a convex lens for both the eyepiece and the objective (unlike the GALILEO telescope of 1610, in which the eyepiece is a concave lens).

In 1613 he married SUSANNE REUTTINGER and of six children only one survived (his first wife died in 1611). When his mother was accused of witchcraft, he took care of her defence and although she was released after 5 years in prison, she died as a result of torture. Earlier, in 1601, KEPLER, on behalf of Emperor RUDOLPH II, had also been charged with witchcraft.

KEPLER had already begun to produce comprehensive astronomical tables from which the position of the sun, the moon, the planets and more than 1000 stars could be read at any time (as well as the times of solar and lunar eclipses).

When JOHN NAPIER invented logarithms in 1614, he immediately realised that they could be used to speed up the complex calculations and he wrote a manual on logarithmic calculation, which contributed greatly to the spread of this method of calculation. The "RUDOLPHIN Tables", published in 1627, remained an important basis for calculations in astronomy for over 200 years.

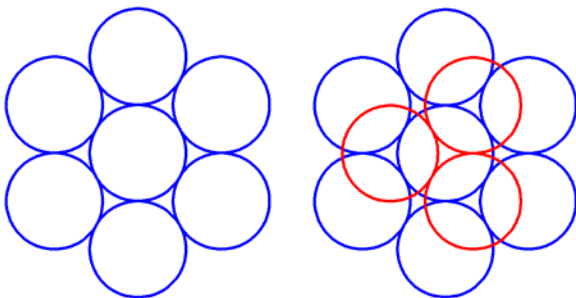


In 1626 he also had to move away from Linz, as he did not want to convert to the Catholic faith. He accepted a position as a mathematician with FÜRST WALLENSTEIN, the famous commander of the 30-year war. When WALLENSTEIN lost his post as the chief general of the Emperor, KEPLER had to move on again. He tried to collect outstanding fees but during a stay in Regensburg he fell ill and died. His grave was destroyed in the turmoil of the 30-year war.

KEPLER's contributions to mathematics were not limited to the novel treatment of conic sections and logarithmic calculations. He discovered the 6-fold symmetry of snow crystals.



He also studied the question of how spheres must be stacked so that they lie as close as possible. KEPLER conjectured that the first layer of spheres is placed on top of each other as shown in the picture; in the next layer (red balls), the spheres are placed in the resulting depressions – every second depression remains empty. For centuries it has not been possible to prove that this so-called hexagonal packing really is the densest ball packing. Computations of the American THOMAS HALES from 1998 have been claimed to give a proof of KEPLER's conjecture.

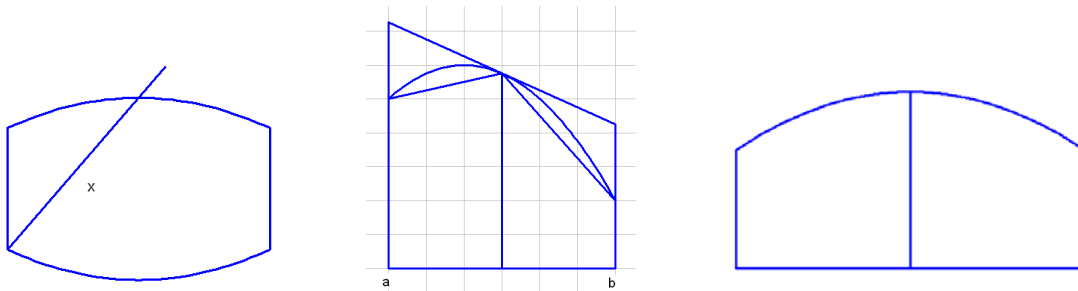


In addition, KEPLER systematically studied polyhedra and classified them. He discovered two stellate bodies which are formed by placing 5-sided pyramids on a dodecahedron or 3-sided pyramids on an icosahedron so that their tips themselves form an icosahedron or dodecahedron.



KEPLER also developed a method for the approximate determination of the volume of barrels and of many other bodies of rotation (*Nova stereometria doliorum vinariorum* - New Stereometry of Wine Barrels). In its derivation he followed the method of ARCHIMEDES, who was earlier able to determine area measurements defined by parabolas.

The reason for his extensive research was an annoying observation when buying wine barrels for his (second) wedding ceremony. The wine merchants determined the contents of the barrels by inserting a rod through the bunghole and measuring the length x to the edges of the bases and then using this to calculate the volume using the formula $V \approx 0.6 \cdot x^3$ whatever the shape of the barrels.



However, the formula known today as *KEPLER's barrel rule* does not indicate the volume of the body of revolution, but the area of an area limited by the graph of a function f on the interval $[a, b]$, whose shape is approximated by a parabolic arc:

$$\int_a^b f(x) dx \approx \frac{b-a}{6} \cdot [f(a) + 4 \cdot f(\frac{a+b}{2}) + f(b)] = \frac{1}{3} \cdot (2S + T)$$

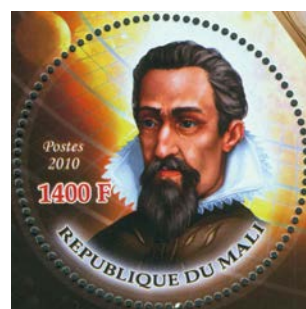
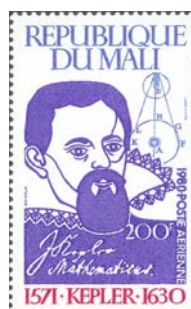
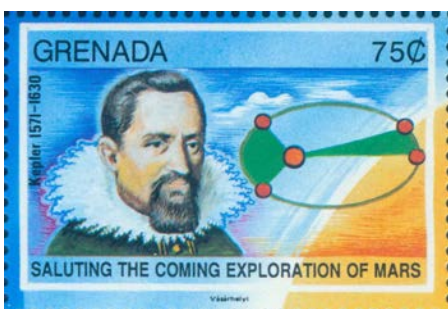
where S is the area of the two quadrilaterals of chords and T that of the tangent quadrilateral. (If f is a quadratic function, then equality applies.)

This results in the following approximate formula for the volume

$$V \approx \pi \cdot \frac{b-a}{6} \cdot [f^2(a) + 4 \cdot f^2(\frac{a+b}{2}) + f^2(b)]$$

and in the symmetrical case of a barrel, the approximate formula is derived from the radii $r = f(a) = f(b)$ and $R = f(\frac{a+b}{2})$ measured at the bottom or in the middle of the barrel and the corresponding circumferences u or U and the barrel height h :

$$V \approx \frac{1}{6} \cdot h \cdot [2\pi r^2 + 4\pi R^2] = \frac{h}{12\pi} \cdot [u^2 + 2U^2]$$





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